Sree Narayana Guru College of Engineering & Technology

CHALAKKODE P.O., KOROM, PAYYANUR, KANNUR-670 307

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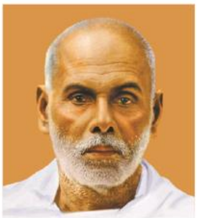

SAMPLES OF ASSIGNMENT

SREE NARAYANA GURU COLLEGE OF ENGINEERING &

TECHNOLOGY

ASSIGNMENT	1	*Academic Year / Semester*	2022-23/01
Subject name with code	221TCE008 STRUCTURAL DYNAMICS	*Branch*	COMPUTER AIDED STRUCTURAL ENGG
Date of Issue	14/11/2022	*Date of submission*	

Q.No	*QUESTIONS*	*Marks*	*CO*	*Level*
1	A vibrating system consist of a mass 5kg, spring of stiffness 120N/m and a damper with a damping co-eff of 5 N-s/m. Determine. a) Damping factor b) Natural frequency & damped frequency c) Logarithmic decrement d) Ratio of 2 successive amplitudes e) No. of cycles after which the initial amplitude is reduced to 25%	10	1	3

CO - Course Outcome [CO]

CO 1: Model and analyse single-degree of freedom systems subjected to free vibration

LEVEL - Bloom's Taxonomy Level

Level 1: Remember

Level 2: Understand

Level 3: Apply

HOD

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ASSIGNMENT	1	Academic Year / Semester	2022-23/01
Subject name with code	221TCE008 STRUCTURAL DYNAMICS	Branch	Computer aided Structural Engg
Date of Issue	14/11/22	Date of submission	21/11/22

ANSWER SCHEME

Q.No		Marks
1.	Damping factor = 0.102	2
	Natural frequency = 4.9 rad/sec	1
	Damped natural frequency = 4.87 rad/sec	1
	Logarithmic decrement = 0.64	2
	Ratio between 2 consecutive amplitudes $\frac{x_1}{x_2} = 1.896$	2
	No. of cycles after 25% reduction = 2.166 ≈ 3 cycles	2

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ASSIGNMENT	2	Academic Year / Semester	2022-23/01
Subject name with code	221TCE008 STRUCTURAL DYNAMICS	Branch	COMPUTER AIDED STRUCTURAL ENGINEERING
Date of Issue	9/12/2022	Date of submission	

Q.No	QUESTIONS	Marks	CO	Level
1	Explain different types of vibration isolation in detail.	10	2	2

CO - Course Outcome [CO]

CO2: Analyse SDOF systems subjected to different dynamic forces and understand the concept of vibration isolation

LEVEL - Bloom's Taxonomy Level

Level 1: Remember

Level 2: Understand

Level 3: Apply

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ASSIGNMENT	2	Academic Year / Semester	22-23/Mtech-S1
Subject name with code	Structural dynamics 221TCE008	Branch	CAS- M.tech
Date of Issue	9/12/22	Date of submission	16/12/22

ANSWER SCHEME

Q.No		Marks
1.	Vibration isolation - General	2
	Passive isolation (Explanation, methods, Significance)	4
	Active isolation (Explanation, methods, Significance)	4

9/12/22

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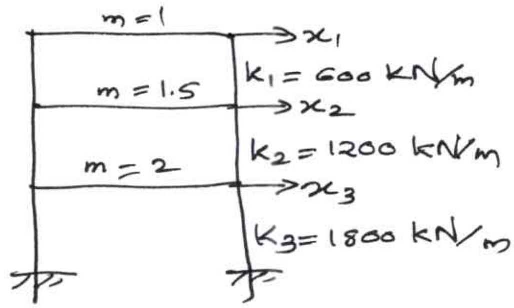
ASSIGNMENT	3	Academic Year / Semester	22-23 / 01
Subject name with code	STRUCTURAL DYNAMICS 221TCE008	Branch	Mtech CAS
Date of Issue	4/01/23	Date of submission	~~0~~ 11/01/23

Q.No	QUESTIONS	Marks	CO	Level
1	Determine the natural frequencies & mode-shapes for the shear building 	10	3	3

CO - Course Outcome [CO]

CO 3: ~~CO~~ : Perform dynamic analysis of MDOF systems

LEVEL - Bloom's Taxonomy Level

Level 1: Remember

Level 2: Understand

Level 3: Apply

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ASSIGNMENT	3	Academic Year / Semester	22-23/01
Subject name with code	STRUCTURAL DYNAMICS 221TCE008	Branch	Mtech CAS
Date of Issue	4/01/23	Date of submission	11/01/23

ANSWER SCHEME

Q.No		Marks
1	Mass matrix	1
	Stiffness matrix	2
	Characteristic equation	
	Natural frequencies $\omega_1 = 14.5$ rad/s $\omega_2 = 31.1$ rad/s $\omega_3 = 46.1$ rad/s	4
	Modes (1) $\begin{Bmatrix} 1 \\ 0.644 \\ 0.3 \end{Bmatrix}$ (2) $\begin{Bmatrix} 1 \\ -0.601 \\ -0.676 \end{Bmatrix}$ (3) $\begin{Bmatrix} 1 \\ -2.57 \\ 2.47 \end{Bmatrix}$	3

4/01/23

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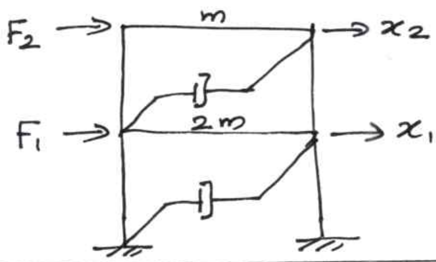
Assignment no.	4	*Academic Year / Semester*	2022-23. / 01.
Subject name with code	STRUCTURAL DYNAMICS 221TCE008	*Branch*	M.TECH CAS
Date of Issue	18/01/23	*Date of submission*	25/01/23

Q.No	*QUESTIONS*	*Marks*	*CO*	*Level*
1	Find the damped vibration response of the 2-storey shear building due to harmonic excitation $\{P(t)\} = \begin{Bmatrix} P_0 \\ 0 \end{Bmatrix} \sin \bar{\omega} t$ & $c = \sqrt{\frac{km}{200}}$ 	10	4	3

CO - Course Outcome [CO]

CO 4 : Perform the analysis of MDOF systems subjected to forced vibration

LEVEL - Bloom's Taxonomy Level

Level 1 : Remember

Level 2 : Understand

Level 3 : Apply

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Assignment no.	4	*Academic Year / Semester*	2022-23 /01
Subject name with code	STRUCTURAL DYNAMICS 221TCE008	*Branch*	M. TECH CAS
Date of Issue	18/01/23	*Date of submission*	25/01/23

ANSWER SCHEME

Q.No		*Marks*
1	Equation of motion & [M], [K], [C], matrices	1
	Solving the characteristic equation and finding the values of natural frequencies & mode shapes $\omega_1 = \sqrt{\frac{k}{2m}}$ $\omega_2 = \sqrt{\frac{2k}{m}}$ $\{\phi_1\} = \begin{Bmatrix} 0.5 \\ 1 \end{Bmatrix}$ $\{\phi_2\} = \begin{Bmatrix} -1 \\ 1 \end{Bmatrix}$	3
	Formation of uncoupled matrices $[M^*]$, $[K^*]$, $[C^*]$, $\{P^*\}$	3
	Calculation of generalized coordinates $q(t)$	2
	Calculation of Displacement response $x(t) = \{\phi\} q(t)$	1

18/01/23

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Assignment no.	5	*Academic Year / Semester*	2022-23 M.TECH S1.
Subject name with code	STRUCTURAL DYNAMICS 221TCE 008	*Branch*	M.TECH CAS
Date of Issue	02/02/23	*Date of submission*	09/02/23

Q.No	*QUESTIONS*	*Marks*	*CO*	*Level*
1	Form the differential equation for axial vibration of rods	10.	5	2.

CO - Course Outcome [CO]

CO 5 : Perform the dynamic analysis of distributed parameter systems

LEVEL - Bloom's Taxonomy Level

Level 1 : Remember

Level 2 : Understand

Level 3 : Apply

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Assignment no.	5	Academic Year / Semester	2022-23/ 01
Subject name with code	STRUCTURAL DYNAMICS 221TCE008	Branch	M.TECH CAS
Date of Issue	02/02/23	Date of submission	09/02/23

ANSWER SCHEME

Q.No		Marks
1	Free body diagram — 3 Stress-strain - axial force relations. — 3 Differential eqn of motion from the free body diagram — 3 General solution — 1	10

02/02/2023

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STRUCTURAL DYNAMICS

ASSIGNMENT - 1

10/10

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SUBMITTED BY,
NIHITHA LOHITHAKSHAN K
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PROBLEM :-

A vibrating system consists of a mass 5 kg, spring of stiffness 120 N/m and a damper with a damping coefficient of 5 N-s/m. Determine :

a) Damping factor

b) Natural frequency of damped vibration

c) Logarithmic decrement

d) The ratio of two successive amplitudes

e) The number of cycles after which the initial amplitude is reduced to 25%.

SOLUTION :-

Given :

$m = 5\,kg$

$c = 5\,Ns/m$

$k = 120\,N/m$

Critical damping coefficient $= c_c = 2\sqrt{km}$

$= 2\sqrt{120 \times 5}$

$= 49\,N\text{-}s/m$

Natural frequency $= \omega_n = \sqrt{k/m}$

$= \sqrt{120/5}$

$= 4.9\,rad/s$

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a) Damping factor $\xi = \frac{c}{c_c}$

$= \frac{5}{49}$

$= \underline{0.102}$

b) Natural Damped frequency $= \omega_d = \omega_n \sqrt{1-\xi^2}$

$= 4.9 \sqrt{1-(0.102)^2}$

$= \underline{4.87 \text{ rad/s}}$

c) Logarithmic decrement. $\delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$

$= \frac{2\pi \times (0.102)}{\sqrt{1-(0.102)^2}}$

$= \underline{0.64}$

d) The ratio between two consecutive amplitudes says x_1/x_2

$\delta = \ln x_1/x_2$

$e^{\delta} = x_1/x_2$

$e^{0.64} = \frac{x_1}{x_2}$

$1.896 = \frac{x_1}{x_2}$

e) Number of cycles after the reduction of 25%.

$\delta = \frac{1}{n} \ln \frac{x_1}{x_2}$

$n = \frac{1}{\delta} \ln \frac{x_1}{x_2}$ $\quad \because x_n = \frac{x_1}{4}$

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$$= \frac{1}{0.64} \ln \left(\frac{x_1}{4} \right)$$

$$= \frac{1}{0.64} \ln 4$$

$= 2.166$ cycles

$= 2$ cycles

?

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STRUCTURAL DYNAMICS SEMINAR

ASSIGNMENT-2

10/10

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VIBRATION ISOLATION

Vibration Isolation is a commonly used technique for reducing or suppressing unwanted vibration in structures and machines. With this technique, the device or system of interest is isolated from the source of vibration through intersection of a resilient member or isolator. There are various types of isolators including metal springs, rubber mounts and pneumatic mounts. Vibration isolation is usually applied in the following two conditions:

(a) The foundation of a vibrating system is protected from large transmitted forces due to HARMONIC EXCITATION or ROTATING UNBALANCED MASS

(b) The vibrating system, which may be a delicate device or instrument, is protected from the motion of its base

In either situation, a TRANSMISSIBILITY of the vibration isolation system at a given frequency, so as to reduce force transmitted to the foundation or to suppress displacement transmitted to the delicate device.

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TYPES OF VIBRATION ISOLATION

1. PASSIVE ISOLATION

'Passive Isolation' refers to vibration isolation or mitigation of vibration by passive technique such as rubber pads or mechanical springs as opposed to "Active vibration Isolation" or "Electronic force cancellation' employing electric power, sensors, actuators and control systems.

A passive vibration isloation system consists of three components, an isolated mass (payload), a spring (k) and a damper (c) and they work as a harmonic oscillator. The payload and spring stiffness defines the natural frequency of the isolating system. While the spring (isolator) reduces floor vibrations from being transmitted to the isolated payload, the damper eliminates the oscillation amplified with in the isolation system. Often passive vibration isolation systems employ a pneumatic springs due to its low resonant frequency characteristics. In many applications, pneumatic system provide outstanding vibration isolation and damping.

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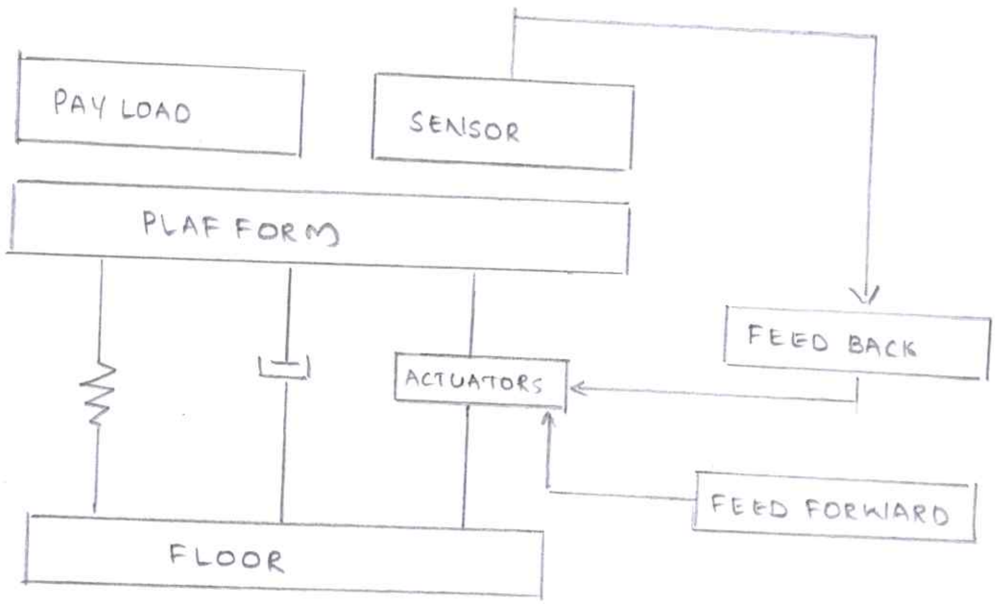
Passive isolation systems are relatively inexpensive and are excellent at mitigating high-frequency vibration. However, their natural resonance is problematic in some applications where low frequency vibration is problematic. Besides the low-frequency resonance, passive vibration isolation systems have a long settling time and are difficult to control.



2. Semi - Active Isolation

Semi active vibration isolators have recieved attention because they consume less power than active devices and controllability over passive systems.

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3. Active Isolation

Active vibration isolation systems consist of feedback and feed-forward control systems with integrated sensors and actuators. These systems isolate the most sensitive equipment from the extremely low-frequency vibration that passive isolation systems amplify at reasonant frequencies. The sensors detect incoming vibration in all six degrees of freedom and a digital controller processes the measured vibration data into digital signals. The controller then sends the signals to the actuators that cancel the vibrations by generating an equal and opposite force. High resolution electronic microscopes and precision - manufacturing tools require active vibration isolation systems when low-frequency vibration is problematic.

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STRUCTURAL DYNAMICS

ASSIGNMENT - 3

10/10

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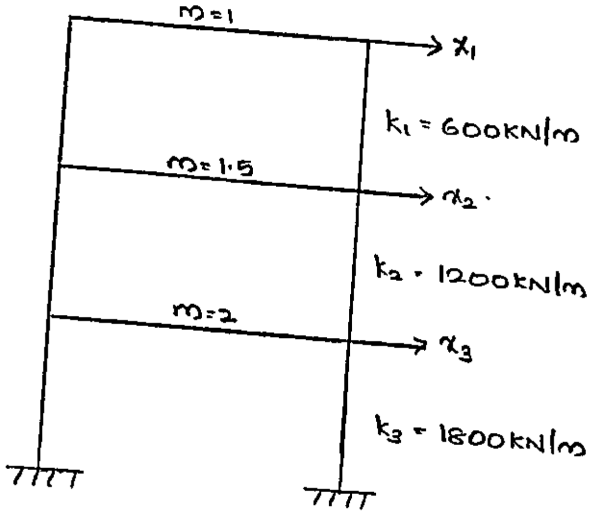
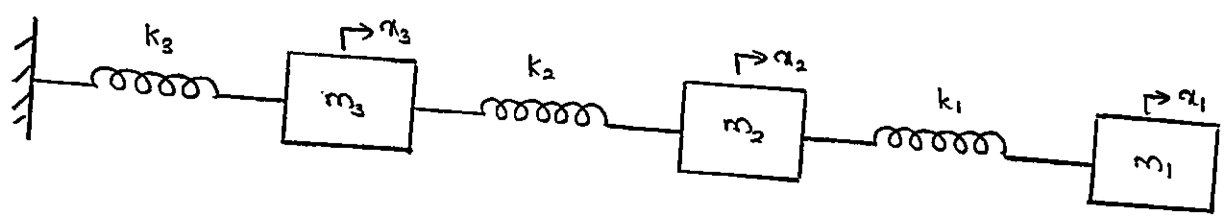
PROBLEM :

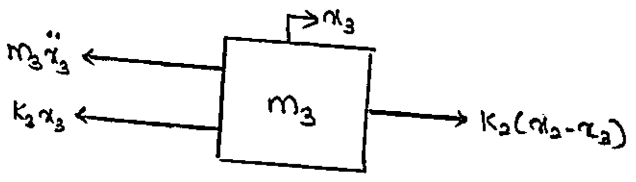
Determine the natural frequencies and the mode shapes for the shear building.

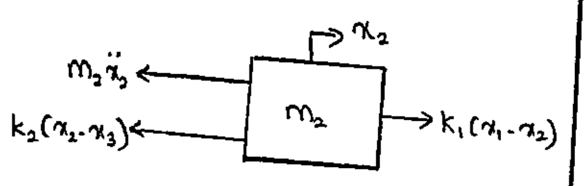


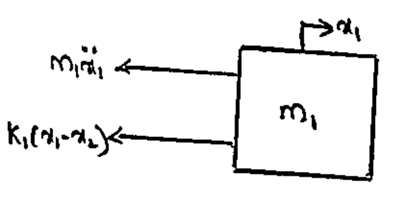
SOLUTION :

This is equivalent to









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Let,

$m_3 = 2$

$m_2 = 1.5$

$m_1 = 1$

Free body diagrams for the equivalent system are ;

$$m_3\ddot{x}_3 + k_3 x_3 - k_2 x_2 + k_2 x_3 = 0$$

$$m_3\ddot{x}_3 + (k_2 + k_3) x_3 - k_2 x_2 = 0 \quad \text{——— (1)}$$

$$m_2\ddot{x}_2 + k_2 x_2 - k_2 x_3 - k_1 x_1 + k_1 x_2 = 0$$

$$m_2\ddot{x}_2 - k_1 x_1 + (k_1 + k_2) x_2 - k_2 x_3 = 0 \quad \text{——— (2)}$$

$$m_1\ddot{x}_1 + k_1 x_1 - k_1 x_2 = 0 \quad \text{——— (3)}$$

Writing eq (1), (2) and (3) into a matrix form

$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{Bmatrix} + \begin{bmatrix} k_1 & -k_1 & 0 \\ -k_1 & k_1+k_2 & -k_2 \\ 0 & -k_2 & k_2+k_3 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \{0\}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1.5 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{Bmatrix} + \begin{bmatrix} 600 & -600 & 0 \\ -600 & 1800 & -1200 \\ 0 & -1200 & 3000 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = 0$$

The characteristic equation is : $|[k] - \omega_n^2 [m]| = 0$

Substituting these values into characteristic equation, we get

$$\left| 600 \begin{bmatrix} 1 & -1 & 0 \\ -1 & 3 & -2 \\ 0 & -2 & 5 \end{bmatrix} - \omega_n^2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1.5 & 0 \\ 0 & 0 & 2 \end{bmatrix} \right| = 0$$

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Dividing by 600

$$\left|\begin{bmatrix} 1 & -1 & 0 \\ -1 & 3 & -2 \\ 0 & -2 & 5 \end{bmatrix} - \frac{\omega_n^2}{600}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1.5 & 0 \\ 0 & 0 & 2 \end{bmatrix}\right| = 0$$

$$\frac{\omega_n^2}{600} = \lambda$$

Let

$$\left|\begin{bmatrix} (1-\lambda) & -1 & 0 \\ -1 & (3-1.5\lambda) & -2 \\ 0 & -2 & (5-2\lambda) \end{bmatrix}\right| = 0$$

Expanding the above determinant, we get

$$\lambda^3 - 5.5\lambda^2 + 7.5\lambda - 2 = 0$$

The roots are:

$$\lambda_1 = 0.351$$

$$\lambda_2 = 1.61$$

$$\lambda_3 = 3.54$$

We know that $\lambda = \frac{\omega_n^2}{600}$

$$\lambda_1 = \frac{\omega_1^2}{600} = 0.351$$

$$\omega_1 = 14.5 \text{ rad/s}$$

$$\lambda_2 = \frac{\omega_2^2}{600} = 1.61$$

$$\omega_2 = 31 \text{ rad/s}$$

$$\lambda_3 = \frac{\omega_3^2}{600} = 3.54$$

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$\omega_3 = 46.1$ rad/s

MODE SHAPES

$$\begin{bmatrix} (1-\lambda) & -1 & 0 \\ -1 & 3-1.5\lambda & -2 \\ 0 & -2 & (5-2\lambda) \end{bmatrix} \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

$$\begin{Bmatrix} -1 \\ 0 \end{Bmatrix} \alpha_1 + \begin{bmatrix} (3-1.5\lambda) & -2 \\ -2 & (5-2\lambda) \end{bmatrix} \begin{Bmatrix} \alpha_2 \\ \alpha_3 \end{Bmatrix} = 0$$

$$\begin{Bmatrix} \alpha_2 \\ \alpha_3 \end{Bmatrix} = -\begin{bmatrix} (3-1.5\lambda) & -2 \\ -2 & (5-2\lambda) \end{bmatrix}^{-1} \begin{Bmatrix} -1 \\ 0 \end{Bmatrix} \alpha_1$$

FIRST MODE

Assume $\alpha_1^{(1)} = 1$

Substituting $\lambda = \lambda_1 = 0.351$

$$\begin{Bmatrix} \alpha_2^{(1)} \\ \alpha_3^{(1)} \end{Bmatrix} = -\begin{bmatrix} 3-1.5\times 0.351 & -2 \\ -2 & 5-2(0.351) \end{bmatrix} \begin{Bmatrix} -1 \\ 0 \end{Bmatrix} (1)$$

$$= \begin{Bmatrix} 0.644 \\ 0.30 \end{Bmatrix}$$

The eigen vector or mode shape corresponding to $\omega_1 = 14.5$ rad/s

$$\{\phi_1\} = \begin{Bmatrix} 1 \\ 0.644 \\ 0.30 \end{Bmatrix}$$

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SECOND MODE

Assume $\alpha_2^{(2)} = 1$

Substituting $\lambda = \lambda_2 = 1.61$

$$\begin{Bmatrix} \alpha_2^{(2)} \\ \alpha_3^{(2)} \end{Bmatrix} = -\begin{bmatrix} (3-1.5\times1.61) & -2 \\ -2 & 5-2(1.61) \end{bmatrix} \begin{Bmatrix} -1 \\ 0 \end{Bmatrix} (1)$$

$$= \begin{Bmatrix} -0.601 \\ -0.676 \end{Bmatrix}$$

Thus, the eigen vector or mode shape corresponding to $\omega_2 = 31.7$ rad/s

$$\{\phi_2\} = \begin{Bmatrix} 1 \\ -0.601 \\ -0.676 \end{Bmatrix}$$

THIRD MODE

Assume $\alpha_1^{(3)} = 1$

Substituting $\lambda = \lambda_3 = 3.54$

$$\begin{Bmatrix} \alpha_2^{(3)} \\ \alpha_3^{(3)} \end{Bmatrix} = -\begin{bmatrix} (3-1.5\times3.54) & -2 \\ -2 & 5-2(3.54) \end{bmatrix} \begin{Bmatrix} -1 \\ 0 \end{Bmatrix} (1)$$

$$= \begin{Bmatrix} -2.57 \\ 2.47 \end{Bmatrix}$$

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The eigen vector or mode shape corresponding to $\omega_3 = 46.1$ rad/s is.

$$\{\phi_3\} = \begin{Bmatrix} 1 \\ -2.57 \\ 2.47 \end{Bmatrix}$$

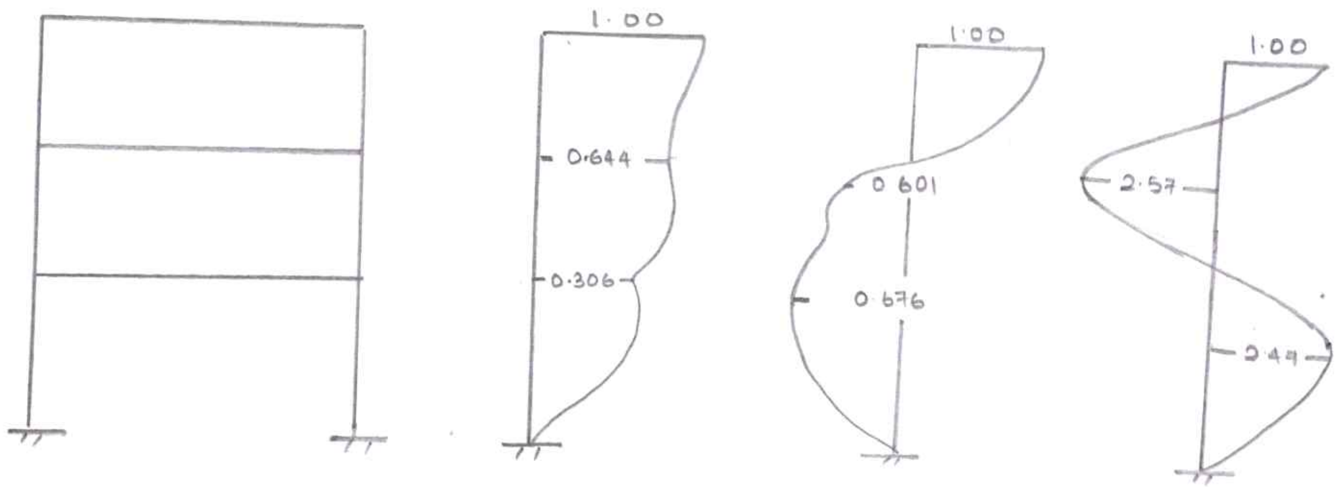
The natural frequencies are,

$\omega_1 = 14.5$ rad/s (fundamental frequency)

$\omega_2 = 31.1$ rad/s

$\omega_3 = 46.1$ rad/s

The shapes of the modes are;



Shear building

Mode 1

$\omega_1 = 14.5$ rad/s

First mode shape

Mode 2

$\omega_2 = 31.1$ rad/s

Second mode shape

Mode 3

$\omega_3 = 46.1$ rad/s

Third mode shape

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STRUCTURAL DYNAMICS

ASSIGNMENT - 4

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SUBMITTED TO,

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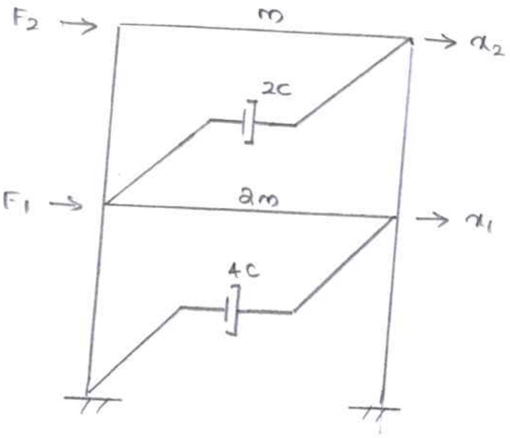
PROBLEM:

Find the damped vibration response of the two storey shear building due to harmonic excitation $\{P(t)\} = \begin{Bmatrix} P_0 \\ 0 \end{Bmatrix} \sin \bar{\omega} t$ and $c = \sqrt{km/200}$



SOLUTION:-

STEP 1: EQUATION OF MOTION

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{P(t)\} \quad \text{— (1)}$$

STEP 2: FIND MODE SHAPE AND NATURAL FREQUENCY BY SOLVING EIGEN VALUE PROBLEM

$$[M] = m\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$$

$$[K] = \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} = k\begin{bmatrix} 3 & -1 \\ -1 & 1 \end{bmatrix}$$

$$[C] = \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix} = c\begin{bmatrix} 6 & -2 \\ -2 & 2 \end{bmatrix}$$

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Obtain Natural modes

$$\left| [k] - \omega_n^2 m [M] \right| = 0$$

$$\left| k \begin{bmatrix} 3 & -1 \\ -1 & 1 \end{bmatrix} - \omega_n^2 m \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \right| = 0$$

$$\left| \begin{bmatrix} 3 & -1 \\ -1 & 1 \end{bmatrix} - \omega_n^2 \frac{m}{k} \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \right| = 0$$

Let $\lambda = \omega_n^2 \frac{m}{k}$

$$\left| \begin{bmatrix} 3 & -1 \\ -1 & 1 \end{bmatrix} - \lambda \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \right| = 0$$

$$\left| \begin{bmatrix} 3 & -1 \\ -1 & 1 \end{bmatrix} - \begin{pmatrix} 2\lambda & 0 \\ 0 & \lambda \end{pmatrix} \right| = 0$$

$$\begin{vmatrix} 3-2\lambda & -1 \\ -1 & 1-\lambda \end{vmatrix} = 0$$

$$(3-2\lambda)(1-\lambda) - 1 = 0$$

$$3 - 3\lambda - 2\lambda + 2\lambda^2 - 1 = 0$$

$$2\lambda^2 - 5\lambda + 2 = 0$$

On solving; $\lambda_1 = 0.5$; $\lambda_2 = 2$.

$\lambda_1 = 0.5$

$$0.5 = \omega_1^2 \frac{m}{k}$$

$$\omega_1^2 = 0.5 \frac{k}{m}$$

$$\omega_1 = \sqrt{\frac{k}{2m}}$$

$\lambda_2 = 2$

$$2 = \omega_2^2 \frac{m}{k}$$

$$\omega_2^2 = 2 \frac{k}{m}$$

$$\omega_2 = \sqrt{\frac{2k}{m}}$$

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FIRST MODE

Substituting these in $\left[[K] - \omega_n^2 [M]\right] \{\phi\} = 0$

$$\left[k\begin{bmatrix} 3 & -1 \\ -1 & 1 \end{bmatrix} - \omega_1^2 m \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}\right] \begin{Bmatrix} \phi_1^{(1)} \\ \phi_2^{(1)} \end{Bmatrix} = 0$$

$$\left[\begin{bmatrix} 3 & -1 \\ -1 & 1 \end{bmatrix} - \frac{\cancel{k}}{2\cancel{m}} \times \frac{\cancel{m}}{\cancel{k}} \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}\right] \begin{Bmatrix} \phi_1^{(1)} \\ \phi_2^{(1)} \end{Bmatrix} = 0$$

$$\left[\begin{bmatrix} 3 & -1 \\ -1 & 1 \end{bmatrix} - \frac{1}{2}\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}\right] \begin{Bmatrix} \phi_1^{(1)} \\ \phi_2^{(1)} \end{Bmatrix} = 0$$

$$\left[\begin{bmatrix} 3 & -1 \\ -1 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 0.5 \end{bmatrix}\right] \begin{Bmatrix} \phi_1^{(1)} \\ \phi_2^{(1)} \end{Bmatrix} = 0$$

$$\begin{bmatrix} 2 & -1 \\ -1 & 0.5 \end{bmatrix} \begin{Bmatrix} \phi_1^{(1)} \\ \phi_2^{(1)} \end{Bmatrix} = 0$$

$$2\phi_1^{(1)} - \phi_2^{(1)} = 0$$

$$2\phi_1^{(1)} = \phi_2^{(1)}$$

$$\frac{\phi_1^{(1)}}{\phi_2^{(1)}} = \frac{1}{2}$$

$$= 0.5$$

$$\left\{\phi_1^{(1)}\right\} = \begin{Bmatrix} 0.5 \\ 1 \end{Bmatrix}$$

SECOND MODE

$$\left[k\begin{bmatrix} 3 & -1 \\ -1 & 1 \end{bmatrix} - \omega_2^2 m \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}\right] \begin{Bmatrix} \phi_1^{(2)} \\ \phi_2^{(2)} \end{Bmatrix} = 0$$

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$$\left[\begin{bmatrix} 3 & -1 \\ -1 & 1 \end{bmatrix} - \frac{2\cancel{k}}{\cancel{m}} \times \frac{\cancel{m}}{\cancel{k}} \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}\right] \begin{Bmatrix} \phi_1^{(2)} \\ \phi_2^{(2)} \end{Bmatrix} = 0$$

$$\left[\begin{bmatrix} 3 & -1 \\ -1 & 1 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}\right] \begin{Bmatrix} \phi_1^{(2)} \\ \phi_2^{(2)} \end{Bmatrix} = 0$$

$$\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \begin{Bmatrix} \phi_1^{(2)} \\ \phi_2^{(2)} \end{Bmatrix} = 0$$

$$-\phi_1^{(2)} - \phi_2^{(2)} = 0$$

$$\frac{\phi_1^{(2)}}{\phi_2^{(2)}} = \frac{1}{-1}$$

$$= -1$$

$$\left\{\phi_2^{(2)}\right\} = \begin{Bmatrix} -1 \\ 1 \end{Bmatrix}$$

STEP 3: MODAL MASS, DAMPING AND STIFFNESS MATRIX

$$[\bar{M}] = \begin{bmatrix} M_1^* & 0 \\ 0 & M_2^* \end{bmatrix} = \{\phi\}^T [M] \{\phi\}$$

$$= \begin{bmatrix} 0.5 & -1 \\ 1 & 1 \end{bmatrix}^T m \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0.5 & -1 \\ 1 & 1 \end{bmatrix}$$

$$= m \begin{bmatrix} 1.5 & 0 \\ 0 & 3 \end{bmatrix}$$

$$\bar{C} = \begin{bmatrix} C_1^* & 0 \\ 0 & C_2^* \end{bmatrix} = \{\phi\}^T [C] \{\phi\}$$

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$$= \begin{bmatrix} 0.5 & -1 \\ 1 & 1 \end{bmatrix}^T c \begin{bmatrix} 6 & -2 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} 0.5 & -1 \\ 1 & 1 \end{bmatrix}$$

$$= c \begin{bmatrix} 1.5 & 0 \\ 0 & 12 \end{bmatrix}$$

$$\overline{k} = \begin{bmatrix} k_1^* & 0 \\ 0 & k_2^* \end{bmatrix} = \{\phi\}^T [k] \{\phi\}$$

$$= \begin{bmatrix} 0.5 & -1 \\ 1 & 1 \end{bmatrix}^T k \begin{bmatrix} 3 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 0.5 & -1 \\ 1 & 1 \end{bmatrix}$$

$$= k \begin{bmatrix} 0.75 & 0 \\ 0 & 6 \end{bmatrix}$$

$$p^* = \{\phi\}^T \{P(t)\}$$

$$= \begin{bmatrix} 0.5 & -1 \\ 1 & 1 \end{bmatrix}^1 \begin{Bmatrix} P_0 \\ 0 \end{Bmatrix} \sin \bar{\omega} t$$

$$= \begin{Bmatrix} 0.5 P_0 \\ -P_0 \end{Bmatrix} \sin \bar{\omega} t$$

STEP 4:

$$q.(t) = \frac{P_0 / k}{\sqrt{(2\xi\beta)^2 + (1-\beta)^2}} \cdot \cos(\bar{\omega} t - \Phi)$$

$$q_1(t) = \frac{0.5 P_0 \sin\bar{\omega} t / k_1^*}{\sqrt{\left(2\xi \frac{\bar{\omega}}{\omega_1}\right)^2 + \left(1 - \frac{\bar{\omega}}{\omega_1}\right)^2}} \cdot \cos(\bar{\omega} t - \Phi)$$

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$$q_1(t) = \frac{0.5\, P_0 \sin\bar{\omega}t / 0.75k}{\sqrt{\left(2\xi\left(\frac{\bar{\omega}}{\sqrt{k/2m}}\right)\right)^2 + \left(1-\left(\frac{\bar{\omega}}{\sqrt{k/2m}}\right)\right)^2}} \cdot \cos(\bar{\omega}t - \bar{\Phi})$$

$$q_2(t) = \frac{P_{20} / k_2^*}{\sqrt{\left(2\xi \frac{\bar{\omega}}{\omega_2}\right)^2 + \left(1-\frac{\bar{\omega}}{\omega_2}\right)^2}} \cdot \cos(\bar{\omega}t - \bar{\Phi})$$

$$= \frac{-P_0 \sin\bar{\omega}t / 6k}{\sqrt{\left[2\xi\left(\frac{\bar{\omega}}{\sqrt{\frac{2k}{m}}}\right)\right]^2 + \left[1-\frac{\bar{\omega}}{\sqrt{2k/m}}\right]^2}} \cdot \cos(\bar{\omega}t - \bar{\Phi})$$

Now,

$$\bar{x}(t) = \{\phi\}_1\, q_1(t) + \{\phi\}_2\, q_2(t)$$

$$= \cos(\bar{\omega}t - \bar{\Phi}) \left[\begin{Bmatrix} 0.5 \\ 1 \end{Bmatrix} \left(\frac{0.5\, P_0 \sin\bar{\omega}t / 0.75k}{\sqrt{\left(2\xi \frac{\bar{\omega}}{\sqrt{k/2m}}\right)^2 + \left(1-\frac{\bar{\omega}}{\sqrt{k/2m}}\right)^2}} \right) + \begin{Bmatrix} -1 \\ 1 \end{Bmatrix} \left(\frac{-P_0 \sin\bar{\omega}t / 6k}{\sqrt{\left(2\xi \frac{\bar{\omega}}{\sqrt{2k/m}}\right)^2 + \left(1-\frac{\bar{\omega}}{\sqrt{2k/m}}\right)^2}} \right) \right]$$

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STRUCTURAL DYNAMICS

ASSIGNMENT 5

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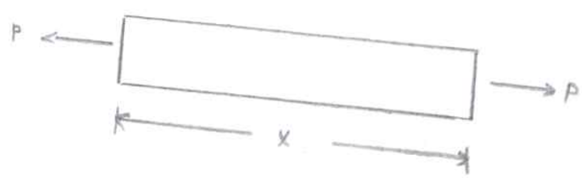
AXIAL VIBRATION OF RODS

Consider a rod having uniform mass per length and cross-section through out the section. So here, the rod is subjected to an axial force P_x which varies from point to point and axial deformation 'u' which is also varying from point to point that is u is a function of x.



At any position x,

$$\varepsilon_x = \frac{\partial u}{\partial x}$$

$$\frac{\text{Stress}}{\text{Strain}} = E$$

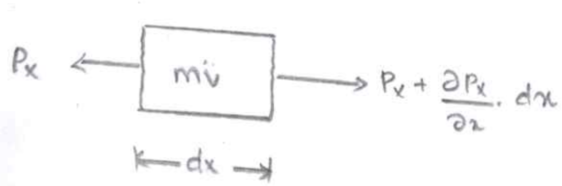
(σ_x) Stress $= E \times \varepsilon_x$

Axial force $= \sigma_x \cdot A$

$$P_x = E \cdot A \cdot \varepsilon_x$$

$$= EA \cdot \frac{\partial u}{\partial x} \quad \text{— ①.}$$

Free body diagram



Equilibrium equation of FBD

$$m\ddot{v} = \cancel{P_x} + \frac{\partial P_x}{\partial x} \cdot dx - \cancel{P_x}$$

Here $m = \rho A dx$

$$\rho A \cancel{dx} \cdot \frac{\partial^2 v}{\partial t^2} = \frac{\partial P_x}{\partial x} \cdot \cancel{dx}$$

$$\rho A \cdot \frac{\partial^2 u}{\partial t^2} = E\cancel{A} \frac{\partial^2 u}{\partial x^2}$$

$$\boxed{\rho \frac{\partial^2 u}{\partial t^2} = E \frac{\partial^2 u}{\partial x^2}}$$

Thus, there are two variables x and t

$$\frac{\partial^2 u}{\partial t^2} = (E/\rho) \frac{\partial^2 u}{\partial x^2}$$

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

Solution is,

$$U(x,t) = \phi(x) \cdot f(t)$$

$$f(t) = A \sin \omega t + B \cos \omega t$$

$$\phi(x) = C \sin(\omega/c)x + D \cos(\omega/c) \cdot x$$

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